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Quantum Computing with light

Quantum optics and single photons have the advantage that, the smallest unit of quantum information, the photon, is potentially free from decoherence. The quantum information encoded stays encoded. The downside is that photons do not interact with each other.

The quantum mechanical plane-wave expansion of the electromagnetic vector potential is usually expressed as

components of 4-vector

Polarization

annihilation op.

$$A^{\mu}(x,t) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} \sum_{j=1,2} \epsilon_j^{\mu}(k)\, \hat{a}_j(k)\, e^{ik x - i\omega t}$$

polarization in coulomb gauge

+ Hermitian Conjugate

$$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle \qquad \hat{a}^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle \qquad \hbar = 1$$

Fock state

k indicates the "spatial mode" and will be denoted by a subscript

Phase shifter: (single-mode)

time dependence is here

$$\hat{a}^{+}_{out} = e^{i\phi \hat{a}^{+}_{in}\hat{a}_{in}}\, \hat{a}^{+}_{in}\, e^{-i\phi \hat{a}^{+}_{in}\hat{a}_{in}} = e^{i\phi}\, \hat{a}^{+}_{in}$$

$$H_\phi = \phi\, \hat{a}^{+}_{in}\hat{a}_{in} \quad \leftarrow \text{interaction Hamiltonian}$$

physically, a phase shifter is a slab of transparent material with a different index of refraction

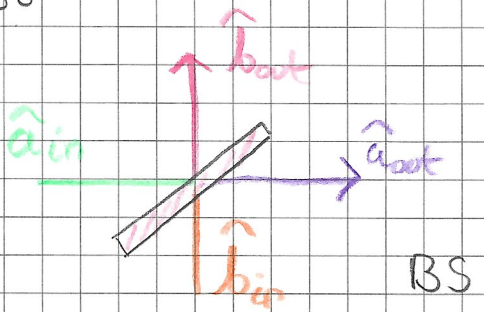
Beam splitter: (BS)

Physically, it consists of a semireflective mirror: when light falls on this mirror, part will be reflected and part will be transmitted.

$$\hat{a}^{+}_{out} = \cos\theta\, \hat{a}^{+}_{in} + i e^{-i\varphi} \sin\theta\, \hat{b}^{+}_{in}$$

$$\hat{b}^{+}_{out} = i e^{i\varphi} \sin\theta\, \hat{a}^{+}_{in} + \cos\theta\, \hat{b}^{+}_{in}$$

$$H_{BS} = \theta e^{i\varphi} \hat{a}^{+}_{in} \hat{b}^{+}_{in} + \theta e^{-i\varphi} \hat{a}_{in} \hat{b}^{+}_{in}$$

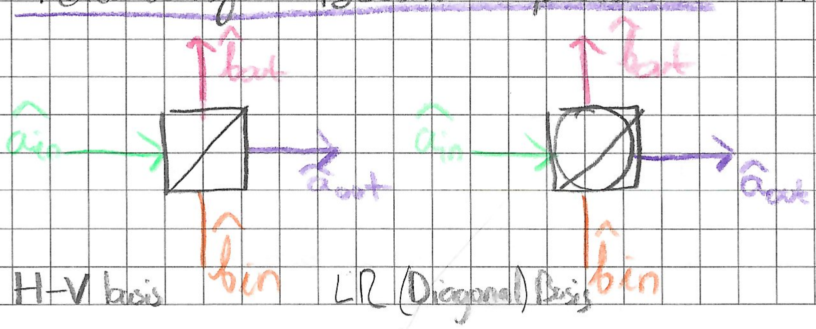


The reflection and transmission coefficients R and T of the beam splitter are

$$R = \sin^2\theta \qquad T = 1 - R = \cos^2\theta$$

relative phase shift $i e^{\pm i\varphi}$ maintains unitary status of H_{BS}

Polarizing Beam Splitter: (PBS)



for the horizontal-vertical (HV) basis

$$\hat{a}_{in,H} \rightarrow \hat{a}_{out,H} \qquad \hat{a}_{in,V} \rightarrow \hat{a}_{out,V}$$
$$\hat{b}_{in,H} \rightarrow \hat{b}_{in,V} \qquad \hat{b}_{in,V} \rightarrow \hat{b}_{out,V}$$

"Linear optics"

"Linear optics" typically denotes the set of optical elements whose interaction Hamiltonian is bilinear in the creation and annihilation operators

$$H = \sum_{jk} A_{jk}\, \hat{a}_j \hat{a}_k$$

This commutes with the total number operator and has the property that a simple mode transformation of creation operators into a linear combination of other creation operators affects only A, but does not introduce terms that are higher order in the creation and annihilation operators

N-port interferometer (Optical Circuits)

Optical circuits can be thought of as a general unitary transformation on N modes, followed by a detection of a subset on the remaining nodes

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The interferometric part of this circuit is also called an N-port interferometer. N ports yield a unitary transformation $\mathcal{U}$ of the spatial field modes a_k with $j, k \in \{1, \ldots, N\}$

linear optics

$$\hat{b}_k \rightarrow \sum_{j=1}^{N} \mathcal{U}_{jk} \hat{a}_j \quad \text{and} \quad \hat{b}_k^{+} \rightarrow \sum_{j=1}^{N} \mathcal{U}_{jk}^{+} a_j^{+}$$

outgoing modes

incoming modes

An su(2) Lie algebra can be formed by

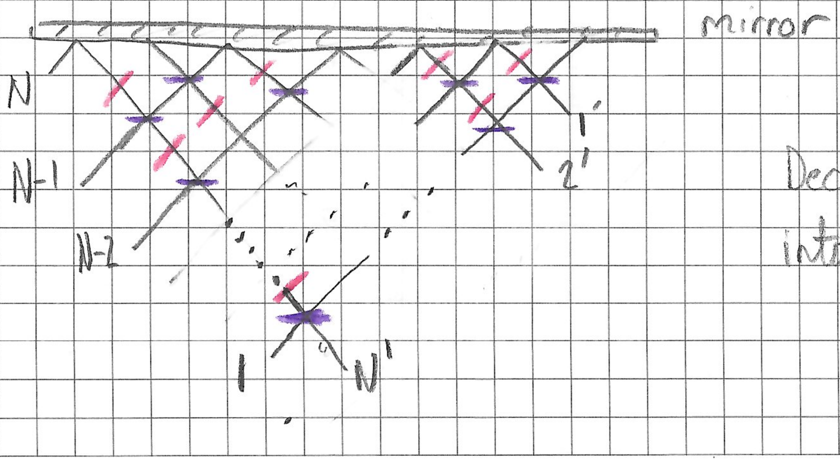
$$\hat{L}_+ = \hat{a}^+ \hat{b}, \quad \hat{L}_- = \hat{a}\hat{b}^+ \quad \hat{L}_0 = (\hat{a}^+\hat{a} - \hat{b}^+\hat{b})/2$$

$$[\hat{L}_0, \hat{L}_\pm] = \pm \hat{L}_\pm \quad [\hat{L}_+, \hat{L}_-] = 2\hat{L}_0$$

Reck et al. (1994) showed that

$$\mathcal{U}(N) \times T_{N,N-1} \times \cdots \times T_{N,1} = \mathcal{U}(N-1) \oplus e^{i\phi}$$

where T_{pq} on modes p and q which corresponds to a beam splitter and phase shifts



Decomposing $\mathcal{U}(N)$ into SU(2) elements

Qubits in linear optics

Formally a qubit is a quantum system that is described by SU(2) symmetry group

In linear optical quantum computing, the qubit of choice is usually taken as

$$|0\rangle_L = |1\rangle \otimes |0\rangle \equiv |1,0\rangle$$
$$|1\rangle_L = |0\rangle \otimes |1\rangle \equiv |0,1\rangle$$

Dual rail qubit

When two modes represent the internal polarization ($|0\rangle_L = |H\rangle$) ($|1\rangle_L = |V\rangle$) this is called a polarization qubit. Dual rail qubit and Polarization qubit are mathematically equivalent, and one can use polarization beam splitters to switch between them.

In order to build a Quantum computer we need both single qubit and two qubit gates

Single qubit gates are straightforward in dual-rail and polarization representations

However two-qubit gates are very difficult

consider the Hadamard gate

$$|H,H\rangle_{ab} \rightarrow \frac{1}{\sqrt{2}}\left(|H,V\rangle_{cd} + |V,H\rangle_{cd}\right)$$

using Bogolubov transformation

$$\hat{a}^\dagger_H \hat{b}^\dagger_H \rightarrow \left(\sum_{k=H,V} \alpha_k \hat{c}^\dagger_k + \beta_k \hat{d}^\dagger_k\right)\left(\sum_{k=H,V} \gamma_k c^\dagger_k + \delta_k d^\dagger_k\right)$$

$$\neq \hat{c}^\dagger_H d^\dagger_V + c^\dagger_V d^\dagger_H$$

for any choice of $\alpha_k, \beta_k, \gamma_k, \delta_k$. The top line is a seperable expression in the creation operators, bottom is an entangled expression.

→ Linear optics alone can not create two qubit gates

If you change representation to $|0\rangle_L = |0\rangle$ $|1\rangle_L = |1\rangle$, then you can make two-qubit gates, but single qubit gates are no longer unitary! (Paris, 2000)

Protocols that solve this problem

Cerf, Adami and Kwiat:

Can be considered as a simulation of a quantum computer. n-qubits are represented by a single photon in 2^n different paths using this protocal a classical version of Groover can be made (Kwiat et al 2000)

Non linear optical element

Such as Kerr medium

$$n_{kerr} = \underbrace{n_0}_{\text{ordinary index}} + \underbrace{\chi^{(3)}}_{\text{constant}} \underbrace{E^2}_{\text{field intensity}}$$

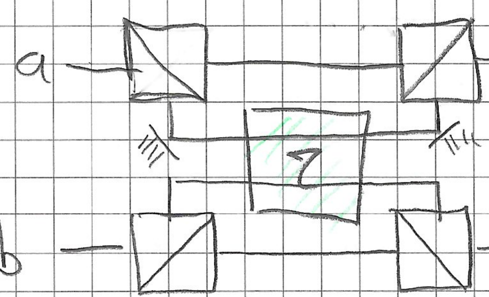
A variation is the cross-Kerr where the phase of a signal shifts as a function of the intensity of a second probe beam

$$H_{kerr} = K \underbrace{\hat{n}_s}_{\text{signal mode}} \underbrace{\hat{n}_p}_{\text{probe mode}}$$

mode transformations are

$$\hat{a}_s \rightarrow \hat{a}_s e^{-i\tau\hat{n}_p} \qquad \hat{a}_p \rightarrow \hat{a}_p e^{-i\tau\hat{n}_s}$$

where $\tau = kt$. When the cross-Kerr medium is placed in one arm of a balanced Mach-Zehnder interferometer, a sufficently strong phase shift τ can switch the field mode.



Single photon CZ gate with Kerr medium

unfortunately τ is two small for practical applications (best: Schmidt and Imamoglu)

Knill, Laflamme, Milburn (KLM)

Physically, deterministic two qubit gates in the polarization and dual-rail representations is not possible, since photons do not interact with each other.

However Bosonic symmetry relation

$$[\hat{a}, \hat{a}^{+}] = 1$$

property can be used to construct an Hadamard gate. Consider a 50:50 beam splitter

$$|1,1\rangle_{a,b} = \hat{a}^{+}\hat{b}^{+}|0\rangle \rightarrow \frac{1}{2}(\hat{c}^{+}+\hat{d}^{+})(\hat{c}^{+}-\hat{d}^{+})|0\rangle_{cd}$$

$$= \frac{1}{2}(\hat{c}^{+2}-\hat{d}^{+2})|0\rangle_{cd}$$

$$= \frac{1}{\sqrt{2}}(|2,0\rangle_{cd} - |0,2\rangle_{cd})$$

photon bunching

Photon bunching is a purely quantum effect where photons pair off

$$|1,1\rangle_{in} \rightarrow_{trans} \cos^2\theta\, |1,1\rangle_{out}$$

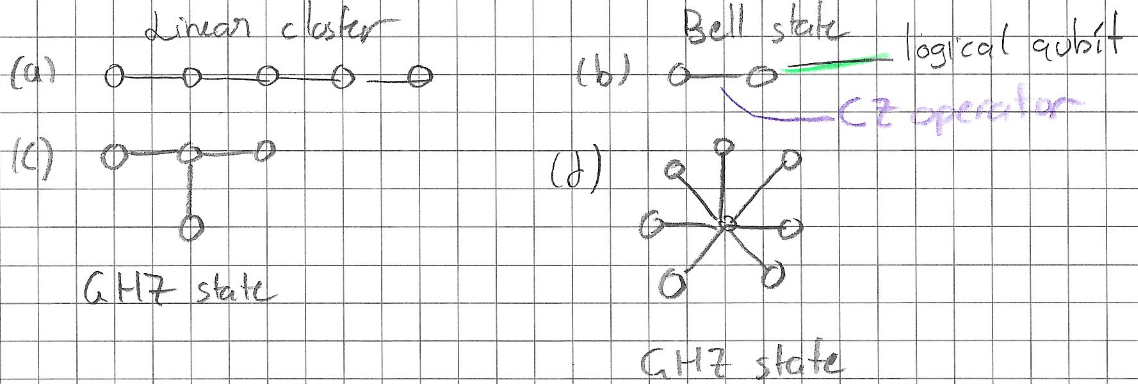
$$|1,1\rangle_{in} \rightarrow_{refl} -\sin^2\theta\, e^{i\varphi}e^{-i\varphi}\, |1,1\rangle_{out}$$

for 50:50 splitter $\cos^2\theta = \sin^2\theta = 1/2$ and two paths cancel exactly irrespective φ. Absence of $|1,1\rangle_{cd}$ is called Ou-Mandel effect. We can use this effect to construct probabilistic Hadamard Gate

probabilistic Nonlinear Sign gates for CZ gate → KLM

Cluster states

Quantum Circuits is not the only model of quantum computing. One alternative is the cluster-state model (Raussendorf and Briegel, 2001) aka, graph-state quantum computing



A cluster state $|C\rangle$ is an eigenstate of a set of commuting operators S_i called the stabilizer generators, hence the graphs represent a family of states

$$S_i |C\rangle = \pm |C\rangle \quad \forall i$$

Given a graph, we can write down stabilizer generators by following

- Every qubit (node) generates an operator S_i
- For each connected neighbor

$$S_q = X_q \; k_j = 1 \otimes Z_j$$

for example (a)

$$S_a = X_a Z_b, \quad S_b = Z_a X_b Z_c$$

$$S_c = Z_b X_c Z_d, \quad S_d = Z_c X_d Z_e$$

$$S_e = Z_d X_e$$

see Nielsen 2006

The computation is done by performing single qubit measurements. Each individual measurement is random, but one seeks correlations. Since this requires renormalisation, each measurement can not be simultaneous.

Boson Sampling

The number of indistinguishable ancilla photons required in linear protocols such as KLM make it very challenging to realise a universal quantum computer with photons

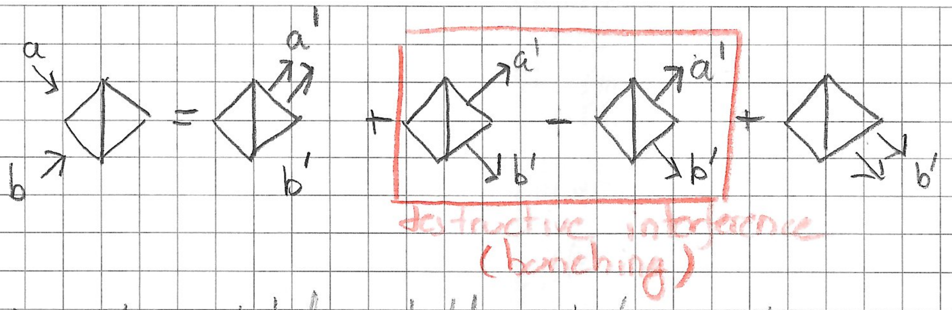
The intermediate quantum models are instead task-specific, but much easier to realise in near term

One important example is "Boson Sampling" intermediate model of quantum computing by Aaronson and Arkhipov.

In general, sampling problems ask for random samples according to a specified probability distribution. The probability distributions and how they are specified depend on the problem class.

This model of computation with non-interacting bosons are not believed to be universal, however, it does not require "measurement", "feed forward" and the problems they attack are at least P-hard.

The bosonic nature of the photons leads to non-classical probability distribution for which there is no known efficient classical sampling algorithm. The simplest case is with two photons (Hong, Ou, Mandel) HOM effect



When two indistinguishable photons enter a 50/50 beam splitter, destructive interference of having both photons either transmitted or reflected leads to bunching such that both photons must be found together in one of the two output modes a' or b'. Therefore the probability of finding two photons in different modes is zero

This is captured by the permanent of the beamsplitter unitary

$$\mathrm{Per}(U)=\sum_{\sigma\in S_m}\prod_{i=1}^{m} U_{i,\sigma(i)}$$

where S_m is the set of all permutations of m elements of $m\times m$ U

for example, the permanent of 2×2 unitary

$U=\begin{pmatrix} a, & b \\ c & d \end{pmatrix}$ is $\mathrm{Per}(U)=ad+bc$

Consider an Input state $|I\rangle=|i_1, i_2, i_3, \dots i_m\rangle$ of n bosons in m states. This state is transformed by $m\times m$ unitary U. The probability of finding the state $|O\rangle=|j_1, j_2, \dots j_m\rangle$ at the output is

$$P_{I,O} = |\langle O| U \otimes U \otimes \cdots \otimes U | I \rangle|^2$$

$$= \frac{|\mathrm{Per}(U_{I,O})|^2}{i_1!\, i_2! \cdots j_1!\, j_2! \cdots j_m!}$$

where $n \times n$ matrix $U_{I,O}$ is defined by first defining U_I by taking i_k copies of the kth column of U, for each $k = 1, \ldots, m$. We then form the $n \times n$ matrix U_{IO} by taking j_k copies of the kth row of U_I for each $k = 1, \ldots m$.

Example

$$U = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix}$$

$$|I\rangle = |1,1,0\rangle$$

$$|O\rangle = |0,1,1\rangle$$

then

$$U_I = \begin{pmatrix} a & b \\ d & e \\ g & h \end{pmatrix}$$

$$U_{I,O} = \begin{pmatrix} d & e \\ g & h \end{pmatrix}$$

therefore the probability of finding this state is

$$P_{I,O} = \frac{|\mathrm{Per}(U_{I,O})|^2}{1!\,1!\,0!\,0!\,1!\,1!} = |dh - eg|^2$$

In 50/50 beam splitter the probability of finding one photon in a' and one photon in b' when there are inputs from a and b is

$$P = |Per(BS)|^2 = \left| Per\begin{pmatrix} T & iR \\ iR & T \end{pmatrix} \right|^2 = |T^2 - R^2|^2 = 0$$

where T and R are Transmission and reflection coefficients

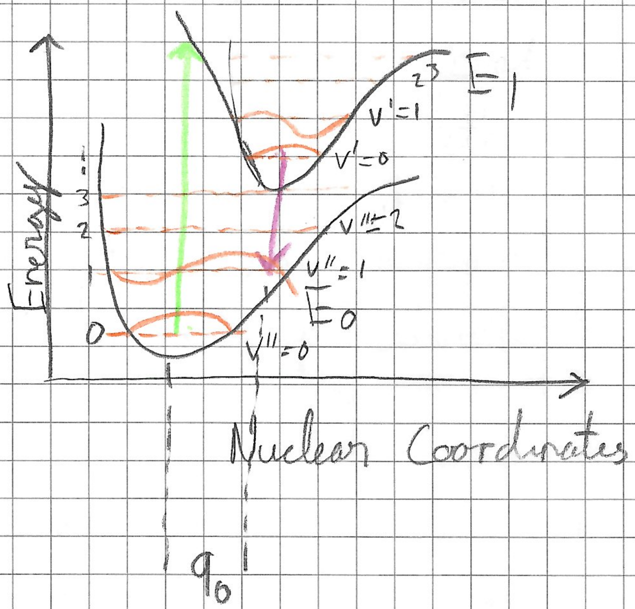
This problem of finding P_{IO} scales exponentially with time. $n \simeq 20$ photons and $m \simeq 400$ modes is already intractable with classical computers

Effects such as "Bosonic birthday paradox" make this even more feasible using optics! For large number of modes $m \gg \sqrt{n}$ the probability of detecting n-photons in n spatially seperated modes as n-fold coincidences dominates, so you don't even need to count the photons hitting the detectors

Application to molecular vibronic spectra

Boson sampling consists of sampling the output distribution of photons obtained from quantum interference inside a linear quantum optical network. Vibronic spectroscopy uses coherent light to electronically excite an ensemble of identical molecules and measures the re-emitted or scattered light. These are formally equivalent together with a step to prepare a non linear step.

Franck - Condon Principle



Since electronic transitions are very fast compared to nuclear motion, vibrational levels are favored when they correspond to a minimal change in the nuclear coordinates

$$|\Psi_{total}(Q,q)\rangle = |\Psi_{nuc}(Q)\rangle\,|\Psi_{el}(Q,q)\rangle$$ (Born-Oppenheimer appr.)

Duschinsky proposed a linear relation to study this effect

$$q' = \mathcal{U}q + d$$

normal coors

displacement (real) vector)

Duschinsky rotation (real) matrix)

final normal coors.

this can be expressed in terms of a modification of the ladder operators[21] as given by

$$\hat{a}'^{+} = \frac{1}{2}(J - (J^{+})^{-1})\hat{a} + \frac{1}{2}(J + (J^{+})^{-1})a^{+} + \frac{1}{\sqrt{2}}\delta$$

where
$$J = \Omega' U \Omega \,, \quad \delta = \hbar^{(-1/2)}\Omega' d$$

$$\Omega' = \mathrm{diag}(\sqrt{\omega_1'}, \ldots \sqrt{\omega_N'})$$
$$\Omega = \mathrm{diag}(\sqrt{\omega_1}, \ldots \sqrt{\omega_N})$$

diag denotes a diagonal matrix and ω are the harmonic angular frequencies of the final and initial states.

The major difference from boson sampling is the appereance of annihilator operators a

Doktorov et al proposed a solution

$$\hat{a}'^{+} = \hat{U}^{+}_{Dok}\,\hat{a}^{+}\,\hat{U}_{Dok}$$

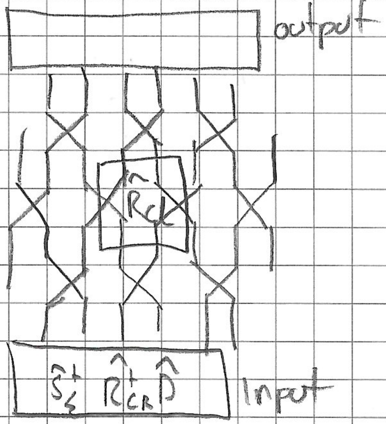
where $\hat{U}_{Dok} = \hat{D}_{\delta/\sqrt{2}}\,\hat{S}^{+}_{\Omega'}\,\hat{R}_{U}\,\hat{S}_{\Omega}$

The transition probability $(\langle m|\hat{U}_{Dok}|n\rangle|^2)$ is called the FC factor (the overlap integral of excited and ground state vibrational levels)

Frank-Condon profile at 0K is obtained with the initial vacuum state $|0\rangle$ as

$$FCP(\omega_{vib}) = \sum_{m}^{\infty} |\langle m|\hat{U}_{Dok}|0\rangle|^2 \, \delta\left(\omega_{vib} - \sum_{k}^{N} \omega_k m_k\right)$$

This can be realised in a photonic circuit as



by means of singular value decomposition

$$J = C_L \, \Sigma \, C_R^{+}$$

Σ → eigenvalues of $J^{+}J$ in singular form

$$\hat{U}_{Dok} = \hat{R}_{C_L} \, \hat{S}^{+}_{\Sigma} \, R^{+}_{C_R} \, \hat{D}_{1/\sqrt{2}\,J^{-1}\delta}$$

computation of this operator is feasible

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